Hybrid Maximum Depth-kNN Method for Real-Time Node Tracking using Multi-Sensor Data

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Introduction

- Sensor node localization and tracking
- Combines two individual location hypothesis functions (LHF)
- Obtained from generalized maximum depth and generalized kNN methods
- Multiple sensors measure visible light, humidity, temperature, acoustics, and link quality
- Lower complexity of maximum depth and outlier rejection ability of kNN method
Motivation

- Univariate data is widely analysed using various statistical moments, like location, scale, skewness, and kurtosis.
- Distance estimation using received signal is erroneous under NLOS conditions.
- Moments are difficult to compute for multivariate data.
- Generally, we analyse a multivariate data on the presumption of a normal distribution.
What is data depth?

- Data depth is an important technique for non-parametric analysis and inference from multivariate data.
- Facilitates systematic way of ordering the multivariate data (center-outward ordering).
- Contours based on data depth analysis provide more intuitive visualisation of distributional properties as compared with statistical moments.
Contributions

- Localization using depth as a measure without any assumption of the distribution of multivariate data
- Developed for the multiple hypothesis testing framework
- We generalize the maximum depth and $k$NN depth based method to account for a vector of multivariate observation under multiple hypothesis
- Generalized maximum depth method requires lesser computation time
- Generalized $k$NN method is robust in the non-line of sight conditions
Notations and Known Definitions

- Let $L$ distinct grid points in the network are denoted by \( \{G_l \equiv (\alpha_l^1, \alpha_l^2, \ldots, \alpha_l^q)\}_{l=0}^{L-1} \), $q$ dimensional coordinate system.

- At each grid, $G_l$, $n_l$ number of training points are collected during offline phase.

- $N$ training observation points, \( O = \{X_1, X_2, \ldots X_N\} \).

- Observations are categorised into $L$ grids namely, $G_0, G_1, \ldots, G_{L-1}$ containing $n_0, n_1, \ldots, n_{L-1}$ points respectively such that \( \sum_{l=0}^{L-1} n_l = N \).
Given an observation vector, $X = \{x_1, x_2, \ldots, x_m, \ldots, x_M\}$, where $x_m \in \mathbb{R}^5$, it is localized to one of the grid point.

Function $I : X \rightarrow Y$ where, $Y \in \{0, 1, \ldots, L - 1\}$ that associates an observation vector $X$ with its corresponding nearest grid $Y$.

Indicator function is denoted by $\mathbb{I}$
A random vector $Z$ in $\mathbb{R}^d$ is said to be symmetric with respect to the following types of symmetricity.

- Centrally symmetric: About $\theta$ if $Z - \theta \overset{d}{=} \theta - Z$, where $\overset{d}{=}$ denotes equal in distribution.
- Angularly symmetric: About $\theta$ if $\frac{(Z - \theta)}{||Z - \theta||} \overset{d}{=} \frac{(\theta - Z)}{||Z - \theta||}$.
- Halfspace symmetric: About $\theta$ if $P(Z \in H \geq 1/2)$ for every closed halfspace containing $\theta$. 
Depth functions

- Used for center outward ordering of points in $\mathbb{R}^d$
- Let $D(.; .) : \mathbb{R}^d \times \mathcal{T} \rightarrow \mathbb{R}^1$ be mapping of depth of points in $\mathbb{R}^d$, whose distribution is given by $\mathcal{T}$.
- Properties: (a) Affine Invariance:

$$D(Ax + b; F_{Ax+b}) = D(x; F_x)$$ (1)

- Maximality at Center:

$$D(\phi; F) = \sup_{x \in \mathbb{R}^d} D(x; F)$$ (2)

- Monotonous with respect to the Deepest Point:

$$D(x; F) \leq D(\phi + \alpha(x - \phi); F) \quad \text{holds for } \alpha \in [0, 1]$$ (3)

- Vanishes at Infinity:

$$D(x; F) = 0 \quad \text{as } \|x\| \rightarrow \infty$$ (4)
Location Depth

- For an observation, \( x \in \mathbb{R}^d \) with respect to a distribution having probability measure \( P \) on \( \mathbb{R}^d \), location depth is defined as the minimum of probability mass of any closed halfspace containing \( x \).

\[
L(x; P) = \inf_{H} \{ P(H) : H \text{ is a closed halfspace, } x \in H \} \quad (5)
\]

- It satisfies all the four desirable properties. Location depth of a point \( x \) with respect to an empirically distributed data set in \( \mathbb{R}^d \) is defined as the minimum fraction of data points lying on either side of any possible hyperplane passing through \( x \).
Simplicial Depth

For an observation, $x \in \mathbb{R}^d$ relative to a probability measure $P$ on $\mathbb{R}^d$, it is defined as the probability that $x$ belongs to a random simplex in $\mathbb{R}^d$.

$$S(x; P) = P(x \in S[X_1, X_2, X_3, \ldots, X_{d+1}])$$ (6)

- Fail to satisfy the monotonicity property and maximality property for centrally symmetric and halfspace symmetric discrete distributions respectively.
- For an empirically distributed data set, it is the ratio of number of simplices containing $x$ to the total number of possible simplices.
Mahalanobis Depth

- The Mahalanobis distance of a point \( x \) from \( O \) with mean \( \mu \) and covariance matrix \( \Sigma \) is defined as

\[
D_m(x) = \sqrt{(x - \mu)\Sigma^{-1}(x - \mu)}
\]  

(7)

- Assuming that second moment of \( O \) exists, Mahalanobis depth of \( x_i \) is defined as

\[
M_i = \left[ 1 + \left( x_i - \bar{X} \right)\Sigma^{-1}\left( x_i - \bar{X} \right) \right]^{-1}
\]

\[
\frac{D_m^2(x_i)}{D_m^2(x)}
\]

(8)

where, \( \bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \) and \( \Sigma = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})(X_i - \bar{X})^\top \).

- Mahalanobis Depth function may fail to satisfy maximality property for angularly symmetric distributions.
For an observation, \( x \), with respect to \( O \) in \( \mathbb{R}^d \), \( L_1 \) depth is defined as one minus average of the unit vectors, directed from \( x \) towards all observations points.

\[
L_1(x, O) = 1 - \left\| \frac{1}{N} \sum_{i=1}^{N} u_i(x) \right\|
\]  

(9)

where, \( u_i(x) = \frac{x - X_i}{\|x - X_i\|} \) denotes the \( i^{th} \) unit vector.

\[
0 \leq L_1(x, O) \leq 1
\]  

(10)

It violates affine invariance property.
For an observation, \( x \in \mathbb{R}^d \) with respect to \( O \), whose distribution is given by \( F \), it is defined as

\[
\Theta(x; F) = \left[ 1 + E_F[v(S[x, X_1, \ldots, X_d])] \right]^{-1}
\]  

(11)

where, \( S[x, X_1, \ldots, X_d] \) represents the closed simplex with vertices \( x \) and random \( d \) points from the \( O \). Expected volume with respect to distribution, \( F \), is represented by \( E_F[v] \).

For an empirically distributed sample in \( \mathbb{R}^2 \), Oja depth of a sensor node position \( x \) is defined as \([?]\)

\[
O(x, F) = \left[ 1 + \frac{1}{\binom{n}{2}} \sum_{\{i,j\} \in \{1,2,\ldots,n\}} \text{Area of triangle with vertices } x, X_i, X_j \right]^{-1}
\]

(12)

Oja depth function may violate affine invariance property.
Generalized Maximum Depth Method

- Binary hypothesis problem,

\[
\hat{h}_D(X; G_0, G_1) = \mathbb{I} \left[ \sum_{i=1}^{M} \mathbb{I} \left[ D(x_i, G_1) > D(x_i, G_0) \right] > \sum_{i=1}^{M} \mathbb{I} \left[ D(x_i, G_0) > D(x_i, G_1) \right] \right] \]

(13)

where, \( n(X, G_l) = \) number of points in observation vector \( X \) having higher depth with respect to \( G_l \) \( \forall \ l = \{0, 1\} \)

- For a multivariate observation vector and multiple hypothesis

\[
\hat{h}_D(X; G_0, G_1, \ldots, G_{L-1}) = \arg\max_l \left[ n(X, G_l) \right] \]

(14)
Generalized $k$NN depth based Method

- Binary hypothesis problem,
  
  $\hat{m}_D^\beta(X; G_0, G_1) = \frac{1}{M} \sum_{j=1}^M \left( \sum_{i=1}^N \mathbb{I}[Y_i = 1] W_i^\beta(x) > \sum_{i=1}^N \mathbb{I}[Y_i = 0] W_i^\beta(x) \right)$

  
  $> \sum_{j=1}^M \left( \sum_{i=1}^N \mathbb{I}[Y_i = 0] W_i^\beta(x) > \sum_{i=1}^N \mathbb{I}[Y_i = 1] W_i^\beta(x) \right)$

  with $W_i^\beta(x) = \frac{1}{K_x^\beta} \mathbb{I}[X_i \in R_x^\beta]$ where $K_x^\beta = \sum_{j=1}^N \mathbb{I}[X_j \in R_x^\beta]$ denotes total number of points in $R_x^\beta$.

- For multiple hypothesis,
  
  $\hat{m}_D^\beta(X; G_0, G_1, \ldots, G_{L-1}) = \arg\max_l \left[ \sum_{j=1}^M \left( \sum_{i=1}^N \mathbb{I}[Y_i = l] W_i^\beta(x) \right) \right]$
Hybrid MD-kNN Method using LHFss

\[ \hat{H}_D(X; G_0, \ldots, G_{L-1}) = \left\{ \begin{array}{ll}
LHF_1 & D(X) > \tau \ast MD \\
LHF_2 & D(X) \leq \tau \ast MD
\end{array} \right. 
\]

Where \( MD \) denotes the maximum depth value obtained from the observation collected at grid points, \( G_i \forall i \).
Statistical Properties of the Hybrid MD-kNN Method

- **Affine Invariance:** **Proof:** Let $X \mapsto Y$ and $Y = AX + b$ represent the affine transformation. $\hat{h}_D(x; G_{1x}, G_{0x})$ denote the maximum depth method for original observation and $\hat{h}_D(y; G_{1y}, G_{0y})$ denote the maximum depth method for the affine-transformed observation.

\[
\hat{h}_D(y; G_{1y}, G_{0y}) = \mathbb{I} \left[ D(y, G_{1y}) > D(y, G_{0y}) \right] \quad (18)
\]

\[
= \mathbb{I} \left[ D(x, G_{1x}) > D(x, G_{0x}) \right]
\]

\[
= \hat{h}_D(x; G_{1x}, G_{0x})
\]

- **Robustness:**
Algorithm for Hybrid MD-kNN for Sensor Localization

1. **Input**: Grid points, \(G_l \triangleq (\alpha^1_l, \alpha^2_l, \ldots, \alpha^q_l)\)\(^{L-1}_{l=0}\) and \(\tau\)

2. **Measurement Acquisition**: Each element of observation vector is a multivariate observation containing visible light, temperature, humidity, acoustic signal and link quality as labels.

3. **Offline Training**: Multivariate observation vectors are recorded at each grid point. At each of the grid, \(G_l\), \(n_l\) training points are collected during offline phase.

4. **Iteration**: Repeat for all the nodes at each time instant.

5. **Online Testing**: Record online observation vector, \(X = \{x_1, x_2, .., x_M\}\), where \(x_m \in \mathbb{R}^5\), at time \(t\) corresponding to a particular sensor node.

6. **Node Localization**: Localize to one of the nearest grid point using the hybrid MD-kNN method in Equation 17.

7. **Termination**: Output, the location of all nodes.
**Experimental Conditions**

*Figure 1:* Figure illustrating the deployments of fifty four sensor nodes in Intel Berkeley Research lab. Reproduced from [?].
Figure 2: Figure illustrating the experimental set-up for firefighter tracking in indoor scenario.
Comparison of Localization Accuracy and Probability of Resolution

Table 1: Comparison of localization accuracy and probability of resolution for various methods using different depth functions, $\tau = 0.2$

<table>
<thead>
<tr>
<th>Depth Functions</th>
<th>Localization Accuracy (m)</th>
<th>Probability of Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MD</td>
<td>MD-kNN</td>
</tr>
<tr>
<td>Location Depth</td>
<td>1.1598</td>
<td>0.8805</td>
</tr>
<tr>
<td>Simplicial Depth</td>
<td>0.8475</td>
<td>0.7556</td>
</tr>
<tr>
<td>Mahalanobis Depth</td>
<td>1.4969</td>
<td>1.3014</td>
</tr>
<tr>
<td>Oja Depth</td>
<td>2.1092</td>
<td>2.1092</td>
</tr>
<tr>
<td>$L_1$ Depth</td>
<td>2.7202</td>
<td>1.8094</td>
</tr>
</tbody>
</table>
Figure 3: Figure illustrating the time complexity of hybrid MD-kNN method for various depth functions. The value of $\beta = \frac{k}{N} = 0.005$. 
Figure 4: Figure illustrating the variation of localization error (m) with threshold of hybrid MD-kNN method for various depth functions. The value of $\beta = 0.005$
Variation of Probability of Resolution with Threshold and Time

Figure 5: Figure illustrating the variation of probability of resolution with threshold and time of hybrid MD-\(k\)NN method, for \(\beta = \frac{k}{N} = 0.005\). The depth functions used are simplicial, location, Mahalanobis, Oja and \(L_1\) from top to bottom respectively on time axis.
Conclusion

- Hybrid MD-$k$NN method for real time sensor node tracking
- Inexpensive and more accurate for sensor node localization
- Future work includes the utilization of functional data with band depth techniques
- Accurately predicts the location of node with malicious sources is also currently being investigated
References I

Monte carlo localization for mobile wireless sensor networks. 

A survey of mobility models for ad hoc network research. 

Anchor-guiding mechanism for beacon-assisted localization in wireless sensor networks. 


Localization for mobile sensor networks. 

Optimal anchor guiding algorithms for maximal node localization in mobile sensor networks. 
Thank You!